# Ad Hoc Wireless Networks: Information-Theoretic Optimal Operating Regimes

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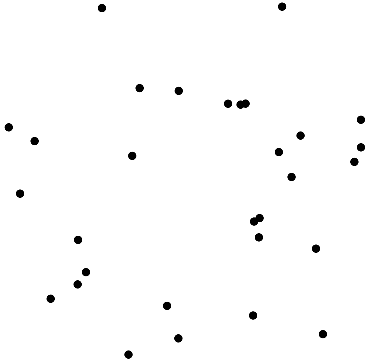
# Model

- $2n$ nodes randomly placed on an area $A(n)$, independently and uniformly



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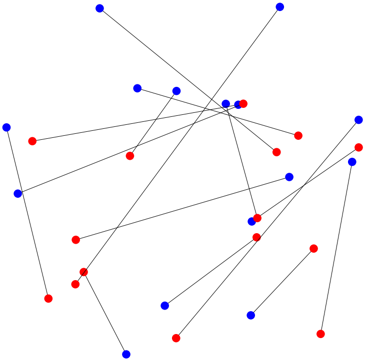

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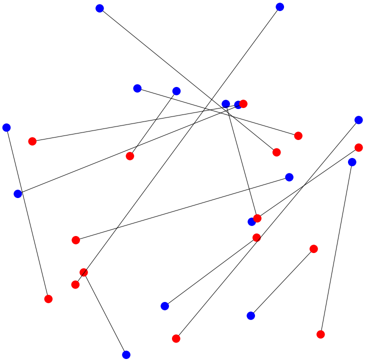
- a set of random connections (logical links) that need to be established, each carrying data at rate $R(n)$
- What is the scaling of the aggregate throughput $T(n) = nR(n)$ with increasing number of nodes $n$?



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$$Y_i = \sum_k H_{ik} X_k + Z_i$$

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- $Z_i$ is additive Gaussian noise with variance $\sigma^2$
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- Related Question:

  What is the "right" scaling to consider for $A(n)$?
  $A(n) = 1$ (dense model)? $A(n) = n$ (extended model)?

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- How can one then talk about scaling laws for wireless networks?

- One needs to choose a model which remains valid in the range of interest (say, e.g., $n \leq 10'000$).

# Another Issue about Scaling Laws: The Example of MIMO systems

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- So if $A \sim n$ (constant density), then $C(n) = O(\sqrt{n})$?
- The question is: for which values of $n$ does this limitation kick in?
- And the answer is: for a highly dense multiple antenna system only! (before this, the assumption of i.i.d. channel coefficients is reasonable)

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- $\Rightarrow$ area limitation? $C \leq \sqrt{10^8} = 10'000 = n$: not a limitation!

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- no final decision for the scaling of $A(n)$ yet

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**Dense Networks ($A(n) = 1$):**

*The optimal aggregate throughput scales as $T(n) = n$.*

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**Extended Networks ($A(n) = n$):**

*The optimal aggregate throughput scales as*

$$T(n) = \begin{cases} n^{2-\alpha/2} & 2 \leq \alpha < 3 \\ n^{1/2} & \alpha \geq 3 \end{cases}$$

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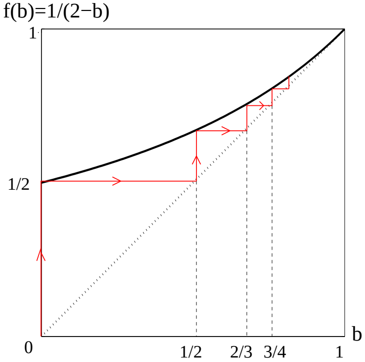
- Key ideas:
  - Distributed MIMO: pool the nodes of the network so as to form virtual multiple antenna arrays.
  - Hierarchical Cooperation: set up cooperation progressively between the nodes, via a hierarchical structure.

# Dense Networks: Hierarchical Cooperation

- Recursive lemma: given a scheme achieving a throughput $n^b$, it is possible to design a new scheme achieving a throughput $n^{1/(2-b)}$ [via distributed MIMO].



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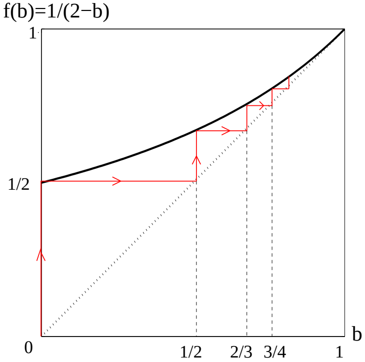

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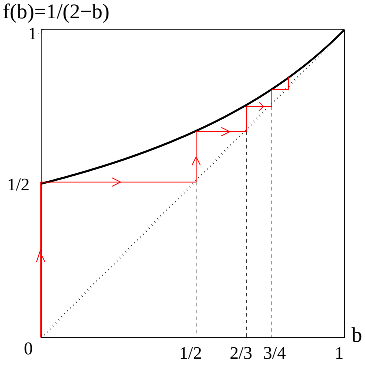


$T(n) = n^{h/(h+1)}$ after $h$ iterations and near network-wide distributed MIMO is achieved (i.e. $n^{h/(h+1)}$ simultaneous long range transmissions!)

# Extended Networks: Proof Idea of the Upper Bound

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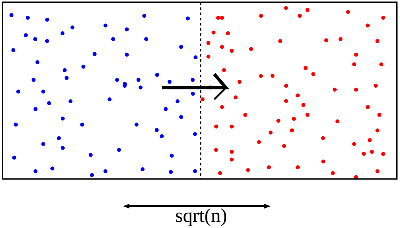
- Cut-set bound argument: the aggregate throughput is limited by the total power received on one side of the cut.



# Extended Networks: Dichotomy

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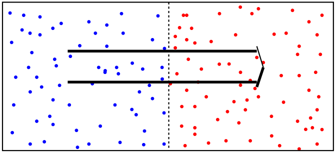

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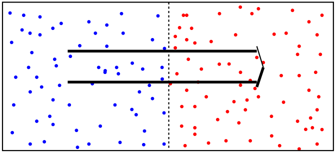
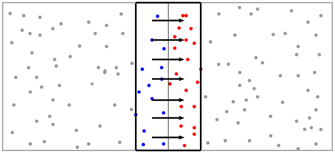


- When $\alpha > 3$, the total received power is $\sqrt{n}$, dominated by transfer between the boundary users: multi-hop is optimal.



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- Intermediate regime: $A(n) = n^{\gamma}$
- For $\alpha > 3$ and $0 < \gamma < 1$, it turns out that neither hierarchical cooperation nor multihop is the optimal strategy (in terms of throughput scaling).
- But a combination of these achieves the optimal throughput scaling:

  "cooperate locally, multi-hop globally"

  (the optimal size of the cooperative clusters being determined by the parameters).

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  - $\text{SNR}_s$ = SNR between nearest neighbour nodes
  - $\text{SNR}_l$ = SNR between two distant clusters of order $n$ nodes
- These parameters are of order:

$$\text{SNR}_s = n^{\beta} \quad \text{and} \quad \text{SNR}_l = n^{1-\alpha/2+\beta}$$

where $\beta = (1-\gamma)\alpha/2$.

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- $\alpha > 3$, $\text{SNR}_l \ll 0$ dB and $\text{SNR}_s \gg 0$ dB: a combination of hierarchical cooperation and multi-hop is optimal

# Summary: Optimal Operating Regimes