A Lower Bound for the Achievable Throughput in Large Random Wireless Networks Under Fixed Multipath Fading

Yoav Nebat
Department of Electrical and Computer Eng.
University of California San Diego
La Jolla, CA 92093, USA
Email: ynebat@ucsd.edu

***Abstract*— We consider the problem of achievable per-node throughput in an extended distributed wireless network where the node locations are random and the channel attenuation between pairs of nodes exhibits independent random multipath fading.**

In [1] a clever protocol construction based on percolation theory was used to show that a per-node throughput of a constant times $\frac{1}{\sqrt{n}}$ bits per second is achievable with probability approaching one as the expected number of nodes in the network, n, becomes large (i.e. w.h.p.), for networks with random node locations under a deterministic channel gain modeling path-loss and absorption.

We use a similar approach to extend the result to a more realistic channel gain model where the channel gains are random due to multipath effects. In particular, we show that a constant times $\frac{1}{\sqrt{n}}\frac{\text{bps}}{\text{node}}$ is also achievable, w.h.p., when the channel gains are random. The result applies to independent, frequency flat fading channel models where the tail probability exhibits an exponential decay (e.g., any mixture of line of sight and Rayleigh, Rice and Nakagami distributions).

I. Introduction

An infrastructure free wireless network where nodes communicate wirelessly over a common wireless medium is considered. Since the landmark work of Gupta and Kumar [2] throughput scaling laws in wireless networks have received considerable attention, e.g. [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13].

A common model in network capacity literature is that of an *extended network*, a network where the area scales with the number of nodes, keeping the node density constant. A permutation demand where each node is associated with two end-to-end traffic flows in the following manner is considered: New traffic for exactly one flow is generated at each node. Similarly, each node is the ultimate destination for a single flow. The origin and destination for each flow are chosen randomly and uniformly, and the metric used to evaluate performance is the per-node throughput.

Variants of this model are also considered in the current literature. While some works allow for a general form of network coding [7], [11], [13], others restrict their discussion to simple multihop packet propagation, allowing point-to-point coding only [1], [2], [14]. Some publications differ on the basis of network geometry, distinguishing between arbitrary networks, where nodes are placed in the plane according to some (possibly favorable) geometry, and random networks, where nodes are placed in the plane at random, often according to a uniform density Poisson point process. In addition, the channel attenuation models may differ. In particular, while most authors account for path-loss and absorption, others account for fixed fading [10], or time-varying fading [13].

Electro-magnetic waves propagate in the wireless medium from the transmit antenna to the receive antenna via multiple paths. The signals received over different spatial paths may add constructively or destructively at the receive antenna, resulting in a variation in the channel attenuation. A simple and accurate model for path attenuation in narrow-band systems is to model the channel attenuation as a product of two coefficients: a random coefficient (the fading coefficient) accounting for multipath effects, and another coefficient accounting for path-loss and absorption effects.

Our work is mainly motivated by the clever protocol construction in [1], where the case of random networks was considered under a pure path-loss and absorption channel gain model and assuming point-to-point coding. The authors apply percolation theory techniques [15], [16] to show that *with high probability* (w.h.p.), i.e., with probability approaching 1 as n approaches infinity, throughput equal to a constant times $\frac{1}{\sqrt{n}}\frac{\text{bps}}{\text{node}}$ is achievable in random networks. Interestingly, the upper bound for the case of arbitrary networks was shown to decay at rate $\frac{1}{\sqrt{n}}$ as n approaches infinity, implying that random network achievable throughput is within a constant factor of arbitrary network throughput when the network is large. Percolation theory has been previously shown to be useful in the study of the connectivity of wireless networks [4], [17] [18], [19], [20]. It is not clear how inclusion of the more realistic random channel model impacts the performance of such a percolation theory based delivery scheme. Under such a model, rather than having a deterministic mapping from distance to channel attenuation, received signal levels are only statistically related to network geometry. Innovative solutions are required to allow for an efficient protocol construction while overcoming the added uncertainty. We address the problem of throughput capacity of fixed, extended random

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networks employing point-to-point coding. In particular, we obtain a lower bound on the achievable per-node throughput, by appropriately adapting the protocol constructed in [1] to a multipath fading environment.

A similar problem was previously addressed in [10] where a per-node throughput of a constant times $n^{-\frac{1}{2}-\varepsilon}\ \forall\ \varepsilon > 0$ was shown to be achieved w.h.p.. The protocol constructed in [10] first divides the network area into a constant times $n^b,\ b < 1$ cells. Subsequently, packets propagate from one cell to the next horizontally and then vertically until they reach the destination.

In this paper we show that under fixed flat fading conditions, if the channel attenuation exhibits power-law path-loss with exponent $\alpha > 2$, or any absorption, which is modeled by exponential attenuation, then throughput equal to a constant times $\frac{1}{\sqrt{n}}\frac{\text{bps}}{\text{node}}$ can be achieved w.h.p.. This proves that, w.h.p., the achievable throughput in such a network, having a random geometry and random channel gains, is no less than a constant factor from the achievable throughput of an arbitrary network under a simple path-loss and absorption channel gain law. Compared to the results in [21], where it was shown that $\frac{1}{\sqrt{n}\log\log n}\frac{\text{bps}}{\text{node}}$ is achievable under similar conditions, for the Rayleigh multipath fading case, this paper further tightens the lower bound and generalizes the result to apply to a more general class of random fading distributions, including mixtures thereof.

The rest of the paper is organized as follows: In Section II we specify the network model, the channel attenuation model and problem formulation. In Section III we provide an outline of the packet forwarding protocol. The protocol is similar to that in [1], with the same three phases and a highway construction based on a mapping to a bond percolation model. In this section, for each of the phases of the protocol, we emphasize new complications and describe the adaptation made to overcome the added uncertainty. Sections IV and V are used to establish theorems that will be needed to prove the result. In Section IV a TDM scheme and a corresponding upper bound on interference is presented. Section V constructs a mapping from the network to a bond percolation model, to show that a highway system rich in crossing paths exists in the network. In Section VI the different protocol phases are described in detail and then analyzed to arrive at our main result. Section VII concludes the paper.

II. Network Model and Problem Formulation

We assume[1] n_1 nodes are placed according to a Poisson point process of unit intensity in a square of area n, defined by $[0, \sqrt{n}] \times [0, \sqrt{n}]$. A permutation traffic demand is assumed. We denote the Euclidean distance between node i and node j by d_{ij}. Each node has peak transmission power P. The power of the signal received from node i at node j is given by $P_i l(i, j)$ where P_i is the power used by node i when it transmits and $l(i, j)$ is the channel attenuation coefficient between node i and node j. We consider channels of the form: $l(i, j) = f_{ij} g_{ij}$ where $g_{ij} = d_{ij}^{-\alpha} e^{-\gamma d_{ij}}$ is a deterministic path-loss and absorption component and f_{ij} models multipath fading effects. We assume that f_{ij} is independent of $\{f_{kl} : (k, l) \notin \{(i, j), (j, i)\}\}$ and $f_{ij} = f_{ji}\ \forall\ i, j$. The distribution of f_{ij} is such that $\forall\ i, j$:

1) f_{ij} is non-negative.
2) $\mathbf{E}[f_{ij}] = 1$.
3) The c.d.f. of f_{ij} has an exponentially decaying tail, i.e.,

$$\Pr\{f_{ij} > f\} \le c_1 e^{-t_e f}\ \forall f > f_0$$

for some real and positive parameters c_1, f_0 and t_e.

We note that Rayleigh, Nakagami and Rice distributions as well as Line Of Sight (LOS) channels (the attenuation of LOS channels is modeled by a deterministic function of the distance, accounting for path-loss and absorption as described above, so that $f_{ij} \equiv 1\ \forall i, j$) all satisfy the above requirements [14, Theorem 6]. Also, the fading coefficients corresponding to the $\frac{n_1(n_1-1)}{2}$ channels in the network may be a mixture of different distributions. We are interested in lower bounding the throughput that is achievable concurrently for all source-destination pairs in the network. Since the geometry and the channel gains in the network are random, no strictly positive throughput can be assured to be supported for any network realization having a finite number of nodes. Therefore, we are mainly interested in events that occur w.h.p.. An event $\mathcal{A}(n)$ is said to occur w.h.p. if $\lim_{n\to\infty} \Pr\{\mathcal{A}(n)\} = 1$. In particular, we restrict our treatment to point-to-point coded transmissions (i.e., multihop) where the capacity of the direct link from i to j is defined as

$$c_{ij} = \log\left(1 + \frac{P_i l(i, j)}{N_0 + \sum_{k, k\neq i} P_k l(k, j)}\right)\frac{\text{bps}}{\text{Hz}}$$

where N_0 is the ambient noise variance, $N_0 > 0$. By constructing an appropriate packet forwarding protocol, we show that w.h.p. a throughput of at least a constant times $\frac{1}{\sqrt{n}}$ bits per second per end-to-end flow in the network is achievable.

III. Overview of The Solution

As mentioned earlier, the protocol construction is an adaptation of the protocol in [1]. While the protocol in [1] applies to networks where the channel gain is a deterministic function of the distance, we account for random multipath fading. In the following we describe the protocol construction and highlight complications due to random fading the required adaptations to the protocol in [1] to overcome this added uncertainty.

W.h.p. a "highway system" can be constructed. The highway system is made of disjoint paths, crossing the network from left to right and top to bottom. A path in the highway system is made of a chain of nodes, where any two neighboring nodes along the chain are no more than a constant distance apart and there exists a *Time Division Multiplexing* (TDM) scheme with a constant number of time slots under which each node along any highway chain has an opportunity to receive a data packet from each of its neighbors in the highway system at least once

[1]The number of nodes, n_1, is random. However, note that by the weak law of large numbers $\lim_{n\to\infty} \Pr\{(1-\varepsilon)n \le n_1 \le (1+\varepsilon)n\} \to 1\ \forall\ \varepsilon > 0$.

in every TDM cycle. For each received packet the *Signal to Interference plus Noise Ratio* (SINR) is assured to exceed a predetermined positive threshold. A horizontal (resp. vertical) path originates no more than a constant distance from the left (resp. bottom) edge of the network and ends within a constant distance from the right (resp. top) edge of the network. The path never crosses any other horizontal (resp. vertical) path in the highway system and never comes closer than a constant distance from the top or bottom (resp. left or right) edges of the network.

The delivery of a packet from source s to destination d is carried in three main phases:

- **Draining:** The packet is sent directly from s to some entry point in the horizontal highway path responsible for draining packets from s. W.h.p., for each node in the network, there exists an entry point which is within some constant times $\log n$ from the source. An appropriate TDM scheme with a constant times $\log^3 n$ slots exists such that, w.h.p., at least a constant times $\frac{1}{\sqrt{n}} \frac{\text{bps}}{\text{node}}$ can be drained.
  In the deterministic channel gain case, as long as the node is within a certain distance from its target horizontal highway path, the node is assured to be able to access the highway through the highway node in the cell exactly North or South of its own cell. Under random multipath fading the channel gain to any specific entry point is not assured to be large enough. By allowing access to one of a large set of potential entry points in the target horizontal highway path, leveraging multiuser diversity, we show that w.h.p., for every node in the network, an appropriate entry point exists (Theorem 3). In addition, due to the random channel gains, the interference bound under TDM in [1] does not apply. We provide a new bound on interference under TDM, applied to the random multipath fading case (Theorem 1). This bound is needed to facilitate proof of the main result.
- **Highway:** The packet propagates in a multi-hop fashion along the horizontal highway path until it reaches the crossing point with the vertical highway path associated with the destination. Then, it proceeds in multi-hop fashion until it reaches the suitable exit point in the vertical highway path (see delivery phase). A TDM scheme with a constant number of slots exists such that, w.h.p., the SINR at any receiving node during the highway phase is no less than a predetermined constant. In addition, the number of links that receive service from any highway path is no more than a constant times $\sqrt{n}$. Therefore, w.h.p., a constant times $\frac{1}{\sqrt{n}} \frac{\text{bps}}{\text{node}}$ can be supported by the highway system. This phase of the protocol happens to be the bottleneck.
  In [1], under TDM with a constant number of time slots, the interference is upper bounded by a constant. Similarly, when the cell side length is a constant, the signal level from a point in any cell to any point in any neighboring cell is lower bounded by a constant. Hence, each transmission to a node in a neighboring cell is assured an SINR above a constant threshold. The mapping to the bond percolation model is then relatively strait forward.
  For the case where the channel gains are random neither is the interference upper bounded by a constant nor is the signal level lower bounded by a constant. We amend this complication by taking into account the channel gains to the points in the neighboring cells and the potential interference at those points before declaring the status of a cell as *open*. Theorem 2 shows that after accounting for the additional constrains, the network is still rich enough in crossing paths.
  Another important feature of the model in [1] is that the event of a cell being open depends only on the event of the existence of at least one point in the cell. The status of the different cells is therefore independent from one cell to the next. This readily available independence is essential to allow the mapping to the bond percolation model. Under fixed multipath fading, the status of the cell depends on different channel gain realizations, as described above. To avoid dependence of the status of a cell on the status of any other cell, a carefully constructed sequential evaluation of the cells' status is suggested. By making sure that a cell is *open* independently of the status of any other cell this construction facilitates the mapping to the bond percolation model. This process is described in detail in Section V.
- **Delivery:** For each node in the network, an exit point exists along an appropriate vertical highway path such that, w.h.p., the distance from the exit point to the destination is no more than a constant times $\log n$. A TDM scheme with a constant times $\log^4 n$ slots can be defined such that w.h.p. it is possible to deliver a constant times $\frac{1}{\sqrt{n}} \frac{\text{bps}}{\text{node}}$.
  Comparing delivery under the deterministic channel gain model in [1] to delivery under the random channel gain model, a similar trend to that of the draining phase arises. Since the destination node, d, is predetermined the source of multiuser diversity is now at the transmit side, i.e., the exit point from the vertical highway path is chosen so that the signal level is above a threshold. The interference is shown to be below the threshold by applying Theorem 1 to the problem at hand.

We formally combine these phases in Section VI to prove the main result.

IV. Interference Under Time Division Multiplexing

In this section we define a TDM scheme that will be used in each of the phases of the packet forwarding protocol. As a preliminary, we divide the $\sqrt{n} \times \sqrt{n}$ area into square cells of constant side length c, as shown in Fig. 1.

Define the following TDM scheme with k^2 time slots: Assign each cell to one of k^2 subsets while making sure the centers of any two cells assigned to the same subset are at

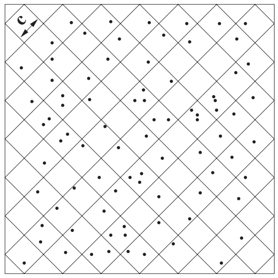

Fig. 1. Dividing the network area into cells of side c.

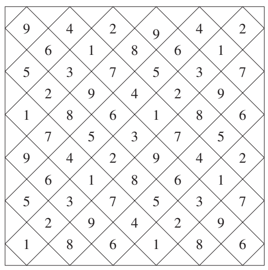

Fig. 2. Subset allocation with $k = 3$.

least kc apart. A subset allocation for $k = 3$ is in Fig. 2.

At time slot $i, i = 1, 2, \cdots, k^2$ up to one node in each cell in subset i transmits to a destination. We limit our discussion to cases where $k = 3(d+1)$ for some positive integer d, and the length of the side of the square within which the destination node is located is $(2d + 1)c$. The square is centered at the center of the cell of the emitting node. Fig. 3 depicts this for $d = 1$ (The grid in Fig. 2 is rotated by 45^o, resulting in Fig. 3).

Each phase of the packet forwarding protocol will use its own TDM scheme. Denote the maximum number of concurrent transmissions in any time slot under the TDM scheme with k^2 time slots by t_k. Since only one node per cell is allowed to transmit in any given slot[2] $t_k = O\left(\frac{n}{k^2}\right)$. Since, by construction of the TDM scheme, there is a one-to-one mapping from transmitters to receivers in each slot, the number of receivers, r_k, is equal to t_k. Denote the maximum number of source-destination pairs whose sources are located in the same cell, across all cells in the network, for the protocol phase under consideration by k_c. Therefore, a schedule with $k^2 k_c$ slots with $O\left(\frac{n}{k^2}\right)$ transmitters and $O\left(\frac{n}{k^2}\right)$ receivers in each slot can be constructed for the phase under consideration. Denote the interference at receiver $j, 1 \leq j \leq r_k$ in slot v, $1 \leq v \leq k^2 k_c$ by $I_d(j, v)$.

Theorem 1 (Interference Bound): Define the event

$$\mathcal{I} \triangleq \left\{ I_d(j,v) \leq K(\sqrt{2}cd)^{-\alpha} e^{-\gamma\sqrt{2}cd} \log n \quad \forall\, j, v \right\}$$

where K is some positive constant. Whenever $\alpha > 2$ or $\gamma > 0$ and as long as $\frac{k_c}{k^2} \leq c_2$, for some constant $c_2 > 0$, $\mathcal{I}$ occurs w.h.p..

Proof: Consider the destination node in Fig. 3. The interfering emitters in the 8 possible interfering cells in Fig. 3 are each more than $\sqrt{2}cd$ away from the destination. Interferers are located in cells whose centers are on the sides of squares with side length $2kci$, $i = 1, 2, \ldots, \left\lfloor \frac{\sqrt{n}}{\sqrt{2}kc} \right\rfloor$. The number of interferers corresponding to the i-th square is at most $8i$ and the distance from the receiver to the closest interferer is more than $\sqrt{2}cdi$ as it is easily verified that in the worst case the distance is

$$(3i - 1)(d + 1)c > \sqrt{2}cdi \quad \forall\, i. \tag{1}$$

Also, note that $m_3 \triangleq \left\lfloor \frac{\sqrt{n}}{\sqrt{2}kc} \right\rfloor$ is $O\left(\frac{\sqrt{n}}{k}\right)$.

Let $I'_d(j, v)$ be an upper bound for $I_d(j, v)$, defined as follows:

1) In the actual network some of the cells defined by the TDM scheme will have no emitting node. Upper bound the interference by adding a fictitious interfering node in each such cell.
2) Consider square $i,\ i = 1, 2, \ldots, \left\lfloor \frac{\sqrt{n}}{\sqrt{2}kc} \right\rfloor$ of side length $2kci$. In general, not all $8i$ cells whose centers are equally spaced on the perimeter of this square are within the $\sqrt{n} \times \sqrt{n}$ network area. Add a fictitious interfering node in each of the cells that is not within the network area.
3) To determine the path-loss and absorption components of the channel attenuation use the lower bound on the distance, as defined in (1).
4) The fading component of the channel attenuation is determined as follows:
   - Real interfering node: use the actual fading coefficient.
   - Fictitious interfering node: generate a fading coefficient at random.
5) Assume all interfering nodes emit at peak power, P.

Accounting for 1–5 above we have

$$I'_d(j, v) \triangleq \sum_{s=1}^{m_3} \sum_{i=1}^{8s} P[\sqrt{2}csd]^{-\alpha} e^{-\gamma\sqrt{2}csd} f^{(v)}_{i+4s(s-1),j}.$$

[2]Notation used throughout the paper: $f = O(g)$ if $\limsup_{n\to\infty} \frac{f(n)}{g(n)} < \infty$; $f = \Omega(g)$ if $g = O(f)$; $f = \Theta(g)$ if $f = O(g)$ and $g = O(f)$. Thus all $O(\cdot)$ results are upper bounds, all $\Omega(\cdot)$ results are lower bounds and $\Theta(\cdot)$ results are sharp scaling estimates.

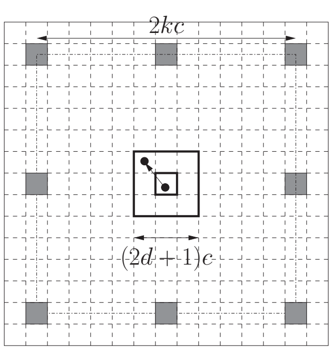

Fig. 3. TDM with $d = 1$: a source-destination pair, destination range and closest interfering cells.

where $\left\{f_{j_1,j_2}^{(v)}\right\}_{j_1=1}^{4m_3(m_3+1)}$ are the fading coefficients from all interferers (real and fictitious) that correspond to slot $v \in \left\{1,2,\ldots,k^2\right\}$ to point j_2.

Note that $I_d(j_1,v_1)$ and $I_d(j_2,v_2)$, are, in general, not independent random variables. There exist several sources of dependency:

- Location of the respective receiving node in the network area and distance from the network's boundary.
- When $v_1 = v_2$, the same nodes interfere with the reception at both j_1 and j_2 (except for node j_2 interfering with j_1's reception and vice versa). Though the random fading components are independent, dependency arises due to interfering node transmit power level and distance dependency reflected in the path-loss and absorption components.

By applying 1 through 5 above to upper bound the interference, all such dependencies are eliminated, so that $I'_d(j_1,v_1)$ and $I'_d(j_2,v_2)$ are independent random variables whenever $(j_1,v_1) \neq (j_2,v_2)$, as shown below

$$\begin{aligned}
&\mathbf{E}[I'_d(j_1,v_1)I'_d(j_2,v_2)]\\
&=\sum_{s_1=1}^{m_3}\sum_{i_1=1}^{8s_1}\sum_{s_2=1}^{m_3}\sum_{i_2=1}^{8s_2}\left[P[\sqrt{2}csd]^{-\alpha}e^{-\gamma\sqrt{2}csd}\right]^2\\
&\quad\cdot\mathbf{E}\left[f_{i_1+4s_1(s_1-1),j_1}^{(v_1)}f_{i_2+4s_2(s_2-1),j_2}^{(v_2)}\right]\\
&=\sum_{s_1=1}^{m_3}\sum_{i_1=1}^{8s_1}P[\sqrt{2}csd]^{-\alpha}e^{-\gamma\sqrt{2}csd}\mathbf{E}\left[f_{i_1+4s_1(s_1-1),j_1}^{(v_1)}\right]\\
&\quad\cdot\sum_{s_2=1}^{m_3}\sum_{i_2=1}^{8s_2}P[\sqrt{2}csd]^{-\alpha}e^{-\gamma\sqrt{2}csd}\mathbf{E}\left[f_{i_2+4s_2(s_2-1),j_2}^{(v_2)}\right]\\
&=\mathbf{E}\left[I'_d(j_1,v_1)\right]\mathbf{E}\left[I'_d(j_2,v_2)\right].
\end{aligned}$$

where the second equality follows since if $j_1 \neq j_2$ these are fading coefficients the correspond to channels terminating at different points. If $v_1 \neq v_2$, the fading coefficients correspond to channels originating at different points.

This observation will be useful during highway construction, which is discussed in the next section.

Define the following event:

$$\mathcal{I}' \triangleq \left\{I'_d(j,v) \leq K(\sqrt{2}cd)^{-\alpha}e^{-\gamma\sqrt{2}cd}\log n \quad \forall\ j,v\right\}$$

Since $I_d(j,v) \leq I'_d(j,v)\ \ \forall\ j,v$ the events $\mathcal{I}$ and $\mathcal{I}'$ are related according to: $\mathcal{I} \supseteq \mathcal{I}'$. It follows that $\Pr\{\mathcal{I}\} \geq \Pr\{\mathcal{I}'\}$.

We now focus on lower bounding $\Pr\{\mathcal{I}'\}$. Consider the following probability

$$\begin{aligned}
p_{I'}(j,v) &\triangleq \Pr\Big\{I'_d(j,v) \leq K(\sqrt{2}cd)^{-\alpha}e^{-\gamma\sqrt{2}cd}\log n\Big\}\\
&=\Pr\Bigg\{\sum_{s=1}^{m_3}\sum_{i=1}^{8s}P[\sqrt{2}csd]^{-\alpha}e^{-\gamma\sqrt{2}csd}f_{i+4s(s-1),j}^{(v)}\\
&\qquad\leq K(\sqrt{2}cd)^{-\alpha}e^{-\gamma\sqrt{2}cd}\log n\Bigg\}\\
&=\Pr\Bigg\{\sum_{s=1}^{m_3}\sum_{i=1}^{8s}s^{-\alpha}\left(e^{-\gamma\sqrt{2}cd}\right)^s f_{i+4s(s-1),j}^{(v)}\\
&\qquad\leq K_1e^{-\gamma\sqrt{2}cd}\log n\Bigg\}
\end{aligned}\tag{2}$$

where $K_1 \triangleq \frac{1}{P}K$. Define $K_2 \triangleq 8\sum_{s=1}^{m_3}s^{1-\alpha}\left(e^{-\gamma\sqrt{2}cd}\right)^{s-1}$. Note that $K_2 < \infty$ when either $\{\alpha > 2, \gamma \geq 0\}$ or $\{\alpha \geq 0, \gamma > 0\}$.

Rewriting (2) we have

$$\begin{aligned}
&p_{I'}(j,v) =\\
&\Pr\Bigg\{\sum_{s=1}^{m_3}\sum_{i=1}^{8s}s^{-\alpha}\left(e^{-\gamma\sqrt{2}cd}\right)^{s-1}e^{-\gamma\sqrt{2}cd}f_{i+4s(s-1),j}^{(v)}\\
&\quad\leq K_3e^{-\gamma\sqrt{2}cd}\left[\sum_{s=1}^{m_3}\sum_{i=1}^{8s}s^{-\alpha}\left(e^{-\sqrt{2}\gamma cd}\right)^{s-1}\right]\log n\Bigg\}
\end{aligned}\tag{3}$$

where $K_3 \triangleq \frac{K_1}{K_2}$.

For any set of random variables, $\{X_i\}_{i=1}^N$, the following always holds

$$\bigcap_{s=1}^{N}\{X_s \leq a_s\} \subset \left\{\sum_{s=1}^{N}X_s \leq \sum_{s=1}^{N}a_s\right\}.$$

Hence, if $\{X_i\}_{i=1}^N$ is a set of independent random variables

$$\Pr\left\{\sum_{s=1}^{N}X_s \leq \sum_{s=1}^{N}a_s\right\} \geq \prod_{s=1}^{N}\Pr\{X_s \leq a_s\}. \tag{4}$$

Applying (4) to (3) we have

$$p_{I'}(j,v) \geq \prod_{s=1}^{m_3}\prod_{i=1}^{8s}\Pr\left\{f_{i+4s(s-1),j}^{(v)} \leq K_3\log n\right\}. \tag{5}$$

As mentioned in Section II, the set of $\frac{n_1(n_1-1)}{2}$ fading coefficients in the network are a mixture of random variables having different distributions. All fading coefficients satisfy the condition $F(f) \equiv \Pr\{f_{ij} \leq f\} \geq 1-c_1e^{-t_ef}\quad \forall\ f \geq f_0$. Hence, for large enough n and $\forall\ l,\ \forall v$

$$\Pr\left\{f_{l,j}^{(v)} \leq K_3\log n\right\} \geq 1-c_1e^{-t_eK_3\log n} = 1-c_1n^{-t_eK_3}.$$

From (5) we get

$$p_{I'}(j,v) \geq (1-c_1n^{-t_eK_3})^{m_4}$$

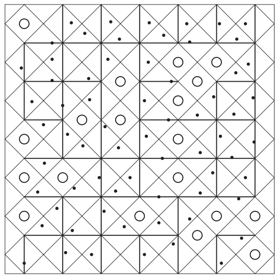

Fig. 4. Remaining edges and points after step 1. Fictitious points are marked by circles.

where $m_4 \triangleq 4m_3(m_3+1)$ is $O\left(\frac{n}{k^2}\right)$.

Since $\left\{I'_d(j,v),\ 1 \le j \le r_k,\ 1 \le v \le k^2k_c\right\}$ is a set of independent, identically distributed random variables, $\Pr\{\mathcal{I}\}$ is lower bounded as follows

$$\Pr\{\mathcal{I}\} \ge \Pr\{\mathcal{I}'\} = \left[p_{I'}(j,v)\right]^{k^2k_cr_k} \ge \left(1 - c_1 n^{-t_eK_3}\right)^{c_3\frac{n^2k_c}{k^2}} \tag{6}$$

for some constant $c_3 > 0$.

When $\frac{k_c}{k^2} \le c_2$, for any constant $c_2 > 0$, the right hand side in (6) tends to one as $n \to \infty$ if $t_eK_3 > 2$. This is satisfied for any $K > \frac{2}{t_e}K_2P$. ■

V. HIGHWAY CONSTRUCTION

Applying the percolation results established in [1] we show the existence of a cluster of nodes forming the wireless backbone. The objective is to formally construct a mesh of paths that can simultaneously carry information across the network at a rate of $\Omega(1)$ bps each. This mesh is referred to as the *highway system*. It is used it to carry packets over most of the distance.

A. Open Cells, Closed Cells and Edges

The highway system will be constructed by mapping the wireless network to a bond percolation model. Note that the probability of a square cell containing at least one point, denoted by p_c, is

$$p_c = 1 - e^{-c^2}.$$

This probability can be adjusted by changing the value of c. The following steps allow us to proceed with the mapping:

1) This step is similar to the construction in [1]. Mark half of the cells as horizontal cells and the other half as vertical cells. In each *horizontal* (resp. *vertical*) cell place an edge diagonally across it horizontally (resp. vertically). For cells having one or more point, arbitrarily select a single point and discard the rest. Discard edges from cells that do not contain a point. Add a fictitious point in the middle of each empty cell. An illustration of the result is in Fig. 4.
2) Discard any horizontal edge if the channel gain from the point in the cell containing that edge to the point in the horizontal cell to its East or to either of the points in the vertical cells to its East are less than a predetermined threshold, T (e.g., in Fig. 5 edge e_i is discarded if l_1, l_2 or l_3 are less than T). Similarly, discard any vertical edge if the channel gain from the point in the cell containing that edge to the point in the vertical cell to its North or to the either of the points in the horizontal cells to its East are less than T (e.g., in Fig. 6 edge e_j is discarded if l_4, l_5 or l_6 are less than T).

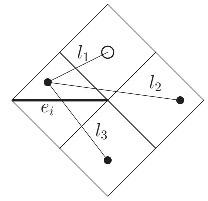

Fig. 5. Deciding whether a horizontal edge should be discarded.

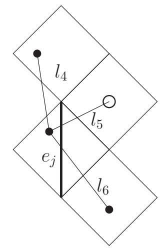

Fig. 6. Deciding whether a vertical edge should be discarded.

3) Consider a TDM scheme as defined in section IV, using $k^2 = 36$ time slots (i.e. $d = 1$). Note that cell i has up to 6 neighboring cells where the edges contained in those cells are directly connected to the edge in cell i (see Fig. 7). Accordingly, let each time slot be further divided into 6 shorter time slots allowing each node to transmit to each such neighbor exactly once in every TDM cycle. Denote the nodes in the cells neighboring cell i by $j_1, j_2, \ldots, j_6$. The edge in cell i is to be discarded unless $I'_1(j_l, v_l) \le I_{max},\ \forall l \in \{l = 1, 2, \ldots, 6\}$ where I_{max} is some predetermined threshold and v_l is the slot intended for transmission from cell i to the cell containing node j_l. An illustration of the network with the remaining edges is in Fig. 8.

We refer to cells within which the edge remains as *open* cells (as opposed to *closed* cells). Note that if two cells are connected by a path (of edges) then there exists a corresponding sequence of single cell hops from the origin to the destination. Each cell visited has a real point in it, the channel gain from

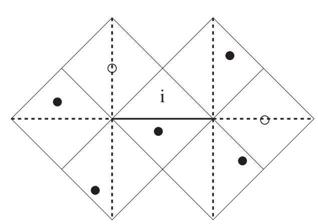

Fig. 7. Cell i and its 6 neighboring cells.

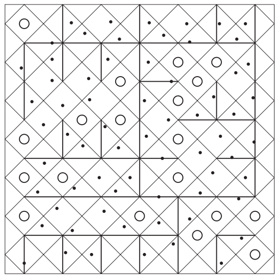

Fig. 8. A network with the remaining edges after steps 2 and 3.

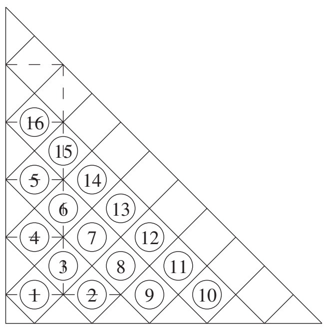

Fig. 9. Identifying open cells.

the point in the previous cell visited to the point in the next cell along the path is at least T, and the interference is less than I_{max} when the TDM scheme defined in step 3 above is used. To see why the signal level is assured to be at least T, consider Fig. 9. If cell 1 is open, the node in it is real, the interference at all neighbors is less than I_{max} and the channels to the points in cells 2 and 3 meet the threshold. We proceed vertically to cell 4. If cell 4 is open, the point in it is real, the interference at all neighbors is less than I_{max} and the channel gains to the points in cells 3, 6 and 7 exceed the threshold. We proceed vertically to cells 5, 16, and so on. Next, consider cell 3. Since connectivity to the points in cells 1 and 4 has already been checked (and is reflected in the status of the corresponding cells), we check connectivity to cells 2, 7 and 6. For cell 6, we only need to check channel gains to points in cells 7, 14 and 15. By proceeding column by column, bottom to top, the new channels are always the channels to the East and to the North, while the channels to the West and South have already been checked when the status of previous cells was determined. An important consequence of this construction is that the event of a cell being open is independent of the status of any other cell in the network, as we show next.

We now bound from below the probability of cell i being open. For cell i to be open three independent conditions must be satisfied. The first requires that the point in the cell is real, occurs with probability p_c independently for all cells. The next two conditions impose requirements on the channel gain to neighboring nodes and the interference at neighboring nodes under TDM. We evaluate the corresponding probabilities next.

For the channel gain requirement to be satisfied at most three different channel gains must be at least T. It is easy to verify that, at most, one of the channels considered corresponds to a cell having a common corner point with the cell under consideration. In such a situation, the distance between any two points, one in each cell, is upper bounded by $2\sqrt{2}c$. Also, at most two channels correspond to cells having a common side with the cell under consideration. The corresponding distance in such a situation is upper bounded by $\sqrt{5}c$ (see Fig. 5 and Fig. 6). Define

$$p_1 \triangleq \Pr\left\{(h(2\sqrt{2}c) > T)\right\},$$
$$p_2 \triangleq \Pr\left\{(h(\sqrt{5}c) > T)\right\},$$

where $h(d_{ij}) \triangleq l(i,j)$. The channel gain requirement is therefore met with probability at least $p_1 p_2^2$, independently for all cells. The probabilities p_1 and p_2 can be set to be as large as desired by setting the value of T low enough.

Next, consider the interference requirement. Whenever $\alpha > 2$ or $\gamma > 0$,

$$\mathbf{E}\left[I_1'(j,v)\right] \leq \sum_{s=1}^{\infty} P\left[\sqrt{2}cs\right]^{-\alpha} e^{-\sqrt{2}\gamma cs} \sum_{i=1}^{8s} \mathbf{E}\left[f_{i+4s(s-1),j}^{(v)}\right]$$
$$= 8\left[\sqrt{2}c\right]^{-\alpha} \sum_{s=1}^{\infty} s^{1-\alpha} e^{-\sqrt{2}\gamma cs} < \infty.$$

Let $\bar{I} \triangleq \mathbf{E}\left[I_1'(j,v)\right]$. By Markov's inequality

$$\Pr\left\{I_1'(j,v) \geq I_{max}\right\} \leq \frac{\bar{I}}{I_{max}}.$$

It follows that

$$p_{I'}(j,v) \triangleq \Pr\left\{I_1'(j,v) \leq I_{max}\right\} \geq \frac{I_{max} - \bar{I}}{I_{max}}.$$

Since $\left\{I_d'(j,v),\ 1 \leq j \leq r_k,\ 1 \leq v \leq k^2 k_c\right\}$ is a set of independent, identically distributed random variables, we can lower bound the probability that none of the interference bounds exceeds I_{max}

$$p_3 \triangleq \prod_{i=1}^{6} p_{I'}(j_i, v) \geq \left(\frac{I_{max} - \bar{I}}{I_{max}}\right)^6.$$

The probability p_3 can be chosen to be as large as desired by choosing I_{max} large enough. It follows that the probability that a cell is open is lower bounded by $p = p_c p_1 p_2^2 p_3 = J \cdot (1 - e^{-c^2})$ where $J \triangleq p_1 p_2^2 p_3$. Each cell's status is independent of the status of the other cells.

B. Bond Percolation Results

Consider bond percolation in a rectangular lattice $\bar{R}$ of size $m_1 \times m_2$, with edge probability p, independently for all edges. Denote the product measure with open edge density p by P_p. Let $\mathcal{A}$ be the event of having at least one open path that crosses $\bar{R}$ from left to right, and let $I_r(\mathcal{A})$ be the event that there exist r edge-disjoint such crossings.

The following lemma is based on a lemma in [15] and follows directly from [1], the reader is referred to [1] for details:

Lemma 1:

$$1 - P_p(I_{\beta m_2}(\mathcal{A})) \leq \left(\frac{p}{p-q}\right)^{\beta m_2} \frac{4}{3} m_1 \left(3(1-q)\right)^{m_2} \quad (7)$$

for any $q < p$ and $\beta > 0$.

Similar to [1], We partition the network into horizontal rectangles, $\bar{R}_n$, of size $\sqrt{n} \times \sqrt{2} c\kappa \log \frac{\sqrt{n}}{\sqrt{2}c}$, where κ is some strictly positive constant. The lattice size of each rectangle is $m \times \kappa \log m$, $m \triangleq \frac{\sqrt{n}}{\sqrt{2}c}$, when edges in the network are mapped to edges in the bond percolation model grid. Since cells are open with probability at least p, the number of disjoint open paths from left to right inside each rectangle is at least as large as the number of crossing paths in a bond percolation model with edge probability p.

Theorem 2 (Number of Crossing Paths): For any constant $\kappa > 0$ and for any c satisfying $c^2 > \log 6 + \frac{2}{\kappa}$, there exists $\beta = \beta(c, \kappa) > 0$, such that w.h.p. there exist $\beta\kappa \log m$ disjoint open paths inside each rectangle $\bar{R}_n$.

Proof: The theorem is identical to that in [1]. However, the probability p has the additional factor J. We show that J can be selected so that the result will still hold.

Recall that $p = J \cdot (1 - e^{-c^2})$ where $J = p_1 p_2^2 p_3$. By selecting T and I_{max} appropriately, we can set J to any value, $J \in (0, 1)$. In particular, choose $J > J_{\min}$ where

$$J_{\min} \triangleq \frac{(1 - 2e^{-c^2})}{(1 - e^{-c^2})^2} = \frac{(1 - 2e^{-c^2})}{(1 - 2e^{-c^2} + e^{-2c^2})}.$$

We next apply Lemma 1. Choose $q = 1 - 2e^{-c^2}$. It follows that $\frac{q}{p} < 1 - e^{-c^2}$, i.e., $\frac{p}{p-q} < e^{c^2}$. Also, $1 - q = 2e^{-c^2}$. Substituting into (7), we have

$$\begin{aligned} 1 - P_p(I_{\beta\kappa \log m}(\mathcal{A})) \leq & \left(e^{c^2}\right)^{\beta\kappa \log m} \frac{4}{3} m \left(6e^{-c^2}\right)^{\kappa \log m} \\ = & \frac{4}{3} m^{(\beta - 1)\kappa c^2 + \kappa \log 6 + 1}. \end{aligned}$$

Since the events occur independently in all $\frac{m}{\kappa \log m}$ rectangles, the probability, Q_β, of having at least $\beta\kappa \log m$ disjoint paths in each rectangle is lower bounded as follows

$$\begin{aligned} Q_\beta = & \left\{P_p\left[I_{\beta\kappa \log m}(\mathcal{A}_n)\right]\right\}^{\frac{m}{\kappa \log m}} \\ \geq & \left(1 - \frac{4}{3} m^{(\beta-1)\kappa c^2 + \kappa \log 6 + 1}\right)^{\frac{m}{\kappa \log m}}. \end{aligned} \quad (8)$$

The right hand side in (8) tends to 1 when $m \to \infty$ if

$$(\beta - 1)\kappa c^2 + \kappa \log 6 + 1 \leq -1.$$

Choosing $c^2 > \log 6 + \frac{2}{\kappa}$, the required condition is satisfied when

$$\beta(c, \kappa) = 1 - \frac{\kappa \log 6 + 2}{\kappa c^2} > 0.$$

■

It follows that the total number of disjoint paths is at least $\beta m = \beta \frac{\sqrt{n}}{\sqrt{2}c}$. Applying a similar construction, by dividing the network into vertical rectangles, we obtain, w.h.p., at least $\beta \frac{\sqrt{n}}{\sqrt{2}c}$ disjoint vertical paths. The highway system is the grid formed by these paths.

VI. The Packet Forwarding Protocol

In this section we define the different phases of the packet forwarding protocol to prove the main result. The bound on the interference intensity under TDM in Section IV (Theorem 1) and the highway construction in Section V (Theorem 2) are the main building blocks used in this section.

A. Draining

We allocate $\frac{1}{3}$ of the time slots in a TDM fashion for this phase. We divide the network area into βm horizontal slabs. The number of edge-disjoint horizontal paths in each rectangle of width $\kappa \log m$ is at least $\beta\kappa \log m$ (Theorem 2). We can arbitrarily choose $\beta\kappa \log m$ disjoint crossing paths in each rectangle to establish a one-to-one mapping between slabs and highway crossing paths in each rectangle. Nodes choose an entry point to the associated crossing path while exploiting spatial diversity such that the received signal power will be above a predetermined threshold. The next theorem makes this precise.

Theorem 3 (Signal Power Bound): W.h.p., for any node in the network there exists an entry point in its associated crossing path that is no more than $2c\kappa \log m$ away for which the received power is at least

$$P \cdot t_2 \cdot (2c\kappa \log m)^{-\alpha} m^{-2\gamma c\kappa}$$

where t_2 is some positive constant.

Proof: From Fig. 10, the vertical distance from a given source to the highway is at most $\sqrt{2}c\kappa \log m$. Thus, within $2c\kappa \log m$ of the source are at least $\kappa \log m$ possible entry points.

The signal level at a corresponding entry point can be assured to exceed a threshold if at least one of the fading coefficients corresponding to channels from the source to all possible entry points exceeds that threshold, i.e., if the maximum of all fading coefficients exceeds the threshold.

For all fading distributions considered, $\forall\ c_4 > 0\ \ \exists\ t_2 > 0$ such that $F_{f_r}(t_2) \equiv \Pr\{f_r \leq t_2\} \leq e^{-c_4}, \quad 1 \leq r \leq \kappa \log m$ where $\{f_r\}_{r=1}^{\kappa \log m}$ are the fading coefficient between the source and corresponding entry points. The probability that the signal level at one of the entry points to the highway is greater than threshold t_2 is therefore lower bounded as follows:

$$\begin{aligned} \Pr\left\{\max_{1 \leq r \leq \kappa \log m} f_r > t_2\right\} = & \left(1 - \prod_{r=1}^{\kappa \log m} F_{f_r}(t_2)\right) \\ \geq & \left(1 - (e^{-c_4})^{\kappa \log m}\right) \\ = & 1 - m^{-\kappa c_4}. \end{aligned}$$

As fading coefficients originating at different sources are independent, the probability that each source in the network will have an entry point such that the signal level is greater

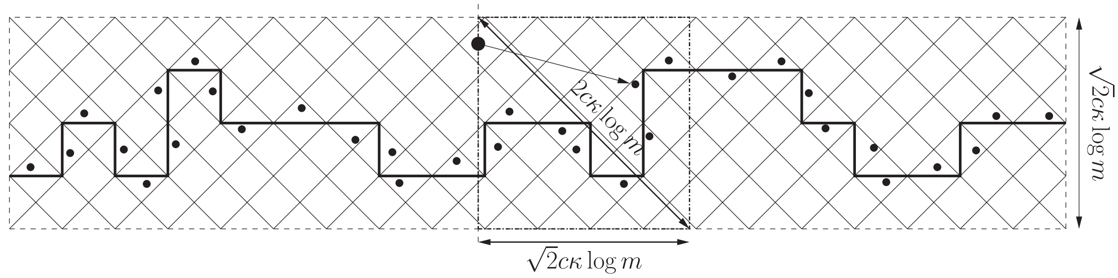

Fig. 10. Draining to the highway.

than t_2 is lower bounded as follows:

$$\left[\Pr\left\{\max_{1\le r\le\kappa\log m} f_r > t_2\right\}\right]^n \ge \left[1-m^{-\kappa c_4}\right]^n = \left[1-m^{-\kappa c_4}\right]^{2c^2m^2}. \quad (9)$$

As $m\to\infty$ the right hand side in (9) tends to 1 as long as $c_4 > \frac{2}{\kappa}$. Consider source node i and assume without loss of generality that the entry point is node j. It follows that the received power at node j is, w.h.p., bounded as follows

$$P_{i,j} = Pf_{i,j}d_{i,j}^{-\alpha}e^{-\gamma d_{i,j}} > P\cdot t_2\cdot(2c\kappa\log m)^{-\alpha}m^{-2\gamma c\kappa}.$$

■

The slots allocated for draining are further distributed into $k^2 = 9(\sqrt{2}\kappa\log m+1)^2$ slots, each serving a set of cells, as described in Section IV. This assures that no more than one source is accessing any entry point at any given time. Note that $9(\sqrt{2}\kappa\log m+1)^2 = 9(d+1)^2$. Hence, $d=\sqrt{2}\kappa\log m$. Applying Theorem 1, $\forall\ j,v$, w.h.p.,

$$I_{\sqrt{2}\kappa\log m}(j,v) \le K(2ck\log m)^{-\alpha}m^{-2\gamma ck}\log n$$

as long as $\frac{k_c}{k^2}\le c_2$. By lemma 3 in [1], w.h.p. there are less than $c^2\log n$ nodes in each cell in network so that $k_c\le c^2\log n$. Therefore, $\frac{k_c}{k^2}\to 0$ as $\log n\to\infty$ and each source has access to the highway at least once in every $3\cdot 9(\sqrt{2}\kappa\log m + 1)^2\cdot c^2\log n < \frac{27}{2}\kappa^2c^2\log^3 n$ slots. Consequently, the SINR for each transmission is at least

$$\frac{Pt_2(2c\kappa\log m)^{-\alpha}m^{-2\gamma c\kappa}}{N_0+K(2c\kappa\log m)^{-\alpha}m^{-2\gamma c\kappa}\log n} = \frac{Pt_2}{N_0(2c\kappa\log m)^{\alpha}m^{2\gamma c\kappa}+K\log n}$$

which allows spectral efficiency

$$\log_2\left(1+\frac{K_4}{(\log m)^{\alpha}m^{2\gamma c\kappa}}\right) \underset{m\to\infty}{\longrightarrow} \frac{K_4\log_2 e}{(\log m)^{\alpha}m^{2\gamma c\kappa}} = \Theta\left((\log n)^{-\alpha}n^{-\gamma c\kappa}\right)$$

where $K_4 \triangleq \frac{Pt_2}{N_0(2c\kappa)^{\alpha}}$.

In summary, w.h.p., $\Theta\left((\log n)^{-\alpha-3}n^{-\gamma cK}\right)\frac{\text{bps}}{\text{node}}$ can be drained to the highway system. Since the highway system can support $\Omega\left(\frac{1}{\sqrt{n}}\right)\frac{\text{bps}}{\text{node}}$ (as shown in Section VI-B), the draining phase will not be a bottleneck if and only if $\gamma c\kappa < \frac{1}{2}$. From Theorem 2 we also require $c^2 > \log 6 + \frac{2}{\kappa}$. A possible solution is: $\kappa = \frac{1}{4\gamma c}$, $c > 4\gamma + \sqrt{16\gamma^2+\log 6}$. If $\gamma = 0$, any $c,\kappa > 0$ satisfying $c^2 > \log 6 + \frac{2}{\kappa}$ can be chosen.

B. Highway

Again, $\frac{1}{3}$ of the time slots are allocated to this phase. Set $d = 1$ (i.e. $k^2 = 9(d+1)^2 = 36$) in the TDM scheme in Section IV. By construction of the highway system, the achievable SINR for each transmission is no less than $\frac{P\cdot T}{N_0+I_{max}}$, implying spectral efficiency equal to $\Omega(1)\frac{bps}{Hz}$. Since each highway node can transmit to each of its neighbors at least once in every $3\cdot 6\cdot 36 = 648$ time slots, $\Omega(1)\frac{bps}{Hz}$ can be concurrently carried over each crossing path. All traffic carried over any horizontal crossing path originates in a single horizontal slab. Similarly, all traffic carried over any vertical crossing path is destined to a single vertical slab. Assume $W\frac{\text{bps/Hz}}{\text{node}}$ are drained to the highway. By [1, lemma 4], w.h.p., no slab contains more than $2c\sqrt{2n}/\beta$ nodes. Hence, any W satisfying $2Wc\frac{\sqrt{2n}}{\beta}\le c_5$, where c_5 is some positive constant, can be supported during the highway phase w.h.p.. Therefore a constant times $\frac{1}{\sqrt{n}}\frac{\text{bps}}{\text{node}}$ can be supported by the highway when transmitting over some fixed bandwidth $0 < W < \infty$.

C. Delivery

The delivery phase is very similar to the draining phase. The packet propagates along a vertical highway crossing path associated with the slab within which the destination is located. Again, there are βm slabs and βm corresponding crossing paths. All nodes along a crossing path are in the same rectangle as the destination, and therefore the horizontal distance to the destination is no more than $\sqrt{2}\kappa c\log m$. Therefore, there are at least $\kappa\log m$ possible exit points within a distance of $2c\kappa\log m$. Theorem 3 applies. Thus, w.h.p., all destinations in the network can be associated with corresponding exit points that are within the allowable distance and can provide a received signal power of at least $P\cdot t_2(2c\kappa\log m)^{-\alpha}m^{-2\gamma c\kappa}$. By allocating the last $\frac{1}{3}$ of the time slots and using a $9(\sqrt{2}\kappa\log m+1)^2$ time slot TDM scheme, the achievable SINR is again, w.h.p., $\Theta\left((\log n)^{-\alpha}n^{-\gamma c\kappa}\right)$ for each destination by applying Theorem 1. Theorem 1 applies since each exit point serves no more than $\frac{2\kappa\log m}{\beta}$ cells, having no more than $c^2\log n$ nodes in each, implying that $k_c < 2\frac{\kappa}{\beta}c^2\log n\cdot\log m$, and

$$\frac{k_c}{k^2} \le \frac{2\kappa c^2\log n\cdot\log m}{9(\sqrt{2}\kappa\log m+1)^2\beta} \underset{m\to\infty}{\longrightarrow} \frac{2c^2}{9\kappa\beta}.$$

Hence, $\Theta\left((\log n)^{-\alpha-4} n^{-\gamma c \kappa}\right) \frac{\text{bps}}{\text{node}}$ can be delivered. Again, choosing $\kappa = \frac{1}{4\gamma c}$, $c > 4\gamma + \sqrt{16\gamma^2 + \log 6}$, $\Omega\left(\frac{1}{\sqrt{n}}\right)$ bps can be delivered to all destinations.

Theorem 4 (Achievable Throughput): A TDM based protocol can be constructed such that, w.h.p., a constant times $\frac{1}{\sqrt{n}}$ bps can be delivered from source to destination, for every source-destination pair in the network.

Proof: From Section VI-A, w.h.p., $\Omega\left(\frac{1}{\sqrt{n}}\right)$ bps can be drained to the highway from each source. From Section VI-B, w.h.p., $\Omega\left(\frac{1}{\sqrt{n}}\right)$ bps can be carried from highway entry point to highway exit point, for every end-to-end link in the network. From Section VI-C, w.h.p., $\Omega\left(\frac{1}{\sqrt{n}}\right)$ bps can be delivered to each destination from the corresponding highway exit point. The result follows. ∎

VII. Conclusion and Future Work

In this paper we address the scaling behavior of the achievable throughput capacity for an extended, distributed wireless network where the nodes are randomly and uniformly placed in a defined area. In particular, we consider the case where the channel attenuation between pairs of nodes is fixed, but exhibits independent random multipath fading.

Motivated by the results in [1] for networks where the channel gain is modeled as a deterministic function of the distance, we extend the $\frac{1}{\sqrt{n}} \frac{\text{bps}}{\text{node}}$ scaling law to the random channel gain model. This new lower bound indicates that the random attenuation coefficients caused by fixed fading inflict a penalty of no more than a constant factor on the per-node throughput, at the limit of large n.

In comparison, the result in [10] implies a polynomial loss factor of n^{ϵ}, where $\epsilon > 0$ can be made small, which results from less efficient interference management, as well as the need for larger cells to assure at least one node exists in each cell, and to allow for sufficient spatial diversity to assure connectivity between neighboring cells. Our construction alleviates these requirements by settling for partial connectivity among neighboring cells. This allows to reduce the cell size to a carefully adjusted constant that assures sufficient connectivity, allowing construction of a "highway system" that is rich enough in crossing paths.

This paper proves that under fixed multipath fading conditions, the throughput scaling is no less than that possible under a simple path-loss and absorption model. The protocol construction presented herein leverages multiuser diversity to overcome the added uncertainty in the channel gain. With random multipath fading, the packet forwarding protocol can not be constructed based only on topology. Spatial reuse and link selection must take the realizations of the random coefficient of the channel gains into account. Whether multiuser diversity can be leveraged further so that a better scaling may be achieved with multipath fading is, to the best of this author's knowledge, an open problem. No result proves the upper bound of $O\left(\frac{1}{\sqrt{n}}\right) \frac{\text{bps}}{\text{node}}$ holds for the fixed random multipath fading case. This problem is currently under investigation.

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