Stability Region of Wireless Networks

C. Bordenave

INRIA & ENS

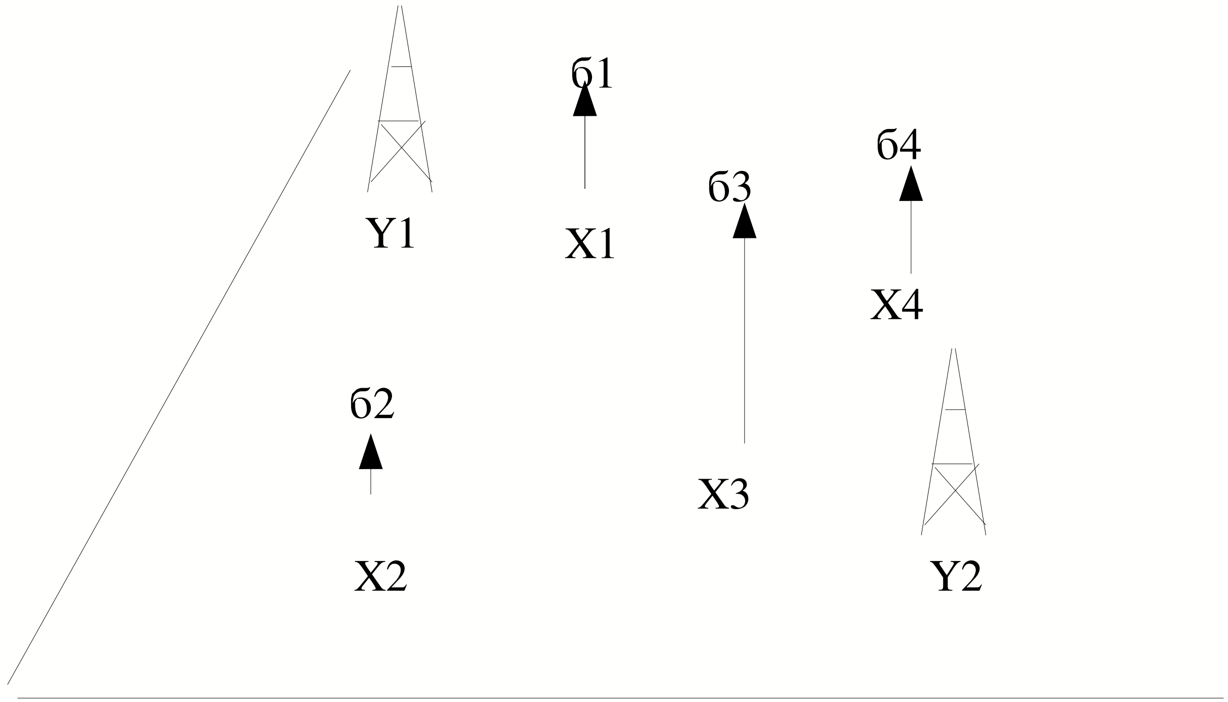
SPASWIN 2005

DATA FLOWS ON THE DOWNLINK

- Users arrive in a wireless network and leave when they have received some data.

- Processing rates vary with positions.

- We consider a large network deployed in the plane.



MODEL DESCRIPTION : ARRIVAL POINT PROCESS

- The n^{th} user arrives at time T_n located at $X_n \in \mathbb{R}^2$ and requires a service of σ_n bits.
- Customers marked point process $\mathbb{R} \times \mathbb{R}^2 \times \mathbb{R}_+$:

$$A = \sum_n \delta_{\{T_n, X_n, \sigma_n\}},$$

- A is time-ergodic (but not necessarily space-stationary)

MODEL DESCRIPTION : ARRIVAL POINT PROCESS

- The intensity measure of A, $\lambda(dx)dt$, is a Radon measure and $\sigma(x) = E^{0,x}(\sigma_{0,x}) \in \mathbb{R}_+$.
  $\sigma(x) =$"mean bits requirement of a typical user located at x"
  For a set B, $\lambda(B) =$"mean number of users arriving by time unit in B"

- For $t \geq T_n$, let $\sigma_n(t)$ by the remaining service of the n^{th} user. The workload at time t is the atomic measure :

$$W_t = \sum_{n:T_n \leq t} \sigma_n(t)\delta_{X_n}.$$

MODEL DESCRIPTION : SERVER STATIONS

- A countable set of server stations with a processing rate $x \to r_j(x)$ such that :

$$\lim_{|x| \to +\infty} r_j(x) = 0.$$

- The server stations are in a random environment. The processing power available at time t is :

$$P_j(t) \geq 0, \quad P_j \text{ is ergodic} \quad \text{and} \quad p_j = EP_j < \infty$$

- We suppose : for all bounded set B, $\quad \sum_j p_j \text{ess} \sup_{x \in B} r_j(x) < +\infty.$

MODEL DESCRIPTION : ADAPTIVE POLICY

Definition 1. *Let $\mathcal{M}$ be the set of Radon measures on $\mathbb{R}^2$, the policy enforced by j^{th} the server station is a mapping :*

$$\begin{array}{ccc} \mathcal{M} & \rightarrow & \mathcal{M} \\ m & \mapsto & \pi_j(m). \end{array}$$

such that :

$$\pi_j(\mathbb{R}^2) = \int_{\mathbb{R}^2} \pi_j(m)(dx) \leq 1$$

and

$\pi_j(m)$ *is absolutely continuous with respect to* m

The policy enforced at time t is : $\pi_j(t) = \pi_j(W_t)$.

MODEL DESCRIPTION : EVOLUTION EQUATION

We can define the dynamic of our queueing system, for all sets B and $t' > t$:

$$W_{t'}(B) = W_t(B) + \int_t^{t'} \int_B \sigma_{s,x} A(ds \times dx) - \sum_j \int_t^{t'} \int_B P_j(s) r_j(x) \pi_j(s)(dx) ds,$$

where : $\int_t^{t'} \int_B \sigma_{s,x} A(ds \times dx) = \sum_n \sigma_n \mathbb{1}(T_n \in [t, t'[) \mathbb{1}(X_n \in B)$.

$$W_{t'} = W_t + \Sigma(t, t') - \sum_j \int_t^{t'} P_j(s) r_j \pi_j(s) ds.$$

TDMA CELLULAR NETWORK

- Server station j is located at $Y_j \in \mathbb{R}^2$.
- To server station j is attached a cell V_j with $V_j \cap V_{j'} = \emptyset$ if $j \neq j'$ and $\cup_j V_j = \mathbb{R}^2$. Such collection of sets $\{V_j\}_j$ will be called a tessellation.
- A server station serves the users in its cell :

$$\forall m \in \mathcal{M}, \quad \pi_j(m)(\mathbb{R}^2 \backslash V_j) = 0.$$

- $P_j(t) = 1$ and a server station serves one user at a time :

$$\pi_j(m) = \delta_{x_j} \text{ or } 0.$$

TDMA CELLULAR NETWORK

- The server stations are constantly emitting at a power S
- Communication channel from x to y with attenuation function $L(x,y)$, (for example $L(x,y) = |x-y|^{-\alpha}, \alpha > 2$).
- Shot Noise : $I(x) = \sum_k L(x, Y_k)$.
- Gaussian white noise at reception of spectral density η,
- Available bandwidth of Δ Hz.
-

$$r_j(x) = \Delta \log_2 \left(1 + \frac{SL(x, Y_j)}{\Delta\eta + S(I(x) - L(x, Y_j))}\right).$$

TDMA CELLULAR NETWORK : FAST FADING

- r_j changes over time. We suppose that $r_j \in \{r_{j,1}, ..., r_{j,k}, ...\}$.
- We divide server station j into servers $\{(j,k)\}_{k\in\mathbb{N}}$.
- $P_{j,k}(t) \in \{0,1\}$, the server stations is switched on or switched off and :

$$\sum_k P_{j,k}(t) = 1.$$

One server station is switched on at any time.

CDMA NETWORK IN MACRODIVERSITY

- The network is in macrodiversity : the server stations may cooperate to send data to users.
- At time t, the signal emitted by the j^{th} base station for the n^{th} user has power $S_{nj}(t)$.
- The system achieves an instantaneous bit rate for user n equal to :

$$R_n(t) = \Delta \log_2 \big(1 + \frac{\sum_j L(X_n, Y_j) S_{nj}(t)}{\Delta\eta + \sum_k L(X_n, Y_k) \sum_{m \neq n} S_{mk}(t)}\big).$$

This condition is only a sufficient condition of achievability of the instantaneous bit rates.

CDMA NETWORK IN MACRODIVERSITY

For CDMA networks :

$$\Delta \log_2(1 + SIR) \approx \frac{\Delta}{\log(2)} SIR$$

Theorem 1. *With the above approximation, if* π *is a policy, there exists a finite power allocation scheme such that :*

$$W_{t'} = W_t + \Sigma(t, t') - \sum_j \int_t^{t'} r_j \pi_j(s) ds,$$

$$r_j(x) = \frac{\Delta}{\log(2)} \frac{L(x, Y_j)}{I(x)}.$$

STABILITY ISSUE

$$W_{t'} = W_t + \Sigma(t,t') - \sum_j \int_t^{t'} P_j(s) r_j \pi_j(s) ds.$$

A policy is stable if there exists a stationary workload satisfying the evolution equation.

The system is stable if there exists a stable policy.

For which class of arrival processes A is the system stable ?

STABILITY REGION OF CELLULAR POLICIES

Let $\{V_j\}_j$, be a tessellation of $\mathbb{R}^2$, a cellular policy with cells $\{V_j\}_j$ is a policy scheme satisfying for all j :

$$\forall m \in \mathcal{M}, \quad \pi_j(m)(\mathbb{R}^d \backslash V_j) = 0.$$

A cellular policy is work-conserving if $m(V_j) > 0$ implies $\pi_j(m)(V_j) = 1$.

Theorem 2. *Any work-conserving cellular policy with bounded cells is stable if :*

$$\forall j, \quad \int_{V_j} \frac{\sigma(x)}{r_j(x)} \lambda(dx) < p_j.$$

If there is a j such that : $\int_{V_j} \frac{\sigma(x)}{r_j(x)} \lambda(dx) > p_j$ then any cellular policy with cells $\{V_j\}_j$ is unstable.

STABILITY REGION : GENERAL CASE

$$\mathcal{F} = \{ \quad f = (f_j)_j \text{ such that,}$$
$$\forall j,\ f_j \geq 0,\ \text{measurable and } \lambda(dx)\text{-a.e. } \sum_j f_j(x) = 1\}.$$

An $f \in \mathcal{F}$ is a partition of the plane.

$\longrightarrow$ Interpretation : A user located at x has a part $f_j(x)$ of its service provided by the j^{th} server station. For a cellular policy : $f_j = \mathbb{1}(V_j)$.

STABILITY REGION

$$\mathcal{N}^s = \{A \in \mathcal{N} : \exists f \in \mathcal{F} \text{ such that} : \forall j, \int_{\mathbb{R}^2} \frac{\sigma(x) f_j(x)}{r_j(x)} \lambda(dx) < p_j\}.$$

$$\bar{\mathcal{N}}^s = \{A \in \mathcal{N} : \exists f \in \mathcal{F} \text{ such that} : \forall j, \int_{\mathbb{R}^2} \frac{\sigma(x) f_j(x)}{r_j(x)} \lambda(dx) \leq p_j\}.$$

Theorem 3. *For the spatial queuing model,*

- *if $A \in \mathcal{N}^s$, then there exists a stable policy,*
- *if there is a stable policy then $A \in \bar{\mathcal{N}}^s$.*

OPTIMAL SPATIAL ALLOCATION

For simplicity, we suppose : $\lambda(dx) = \lambda(x)dx$. Stability relies on the value of :

$$\begin{aligned} \rho &= \inf_{f \in \mathcal{F}} \sup_{j \in \mathbb{N}} \frac{1}{p_j} \int_{\mathbb{R}^2} \frac{\sigma(x) f_j(x)}{r_j(x)} \lambda(x) dx \\ &= \inf_{f \in \mathcal{F}} \rho(f). \end{aligned}$$

If $\rho < 1$, the system is stable, if $\rho > 1$, the system is unstable.

OPTIMAL SPATIAL ALLOCATION

Let

$$\mathcal{F}^* = \{f \in \mathcal{F} : \rho(f) = \rho\}.$$

Proposition 1. *There is an* $f \in \mathcal{F}^*$ *such that :*

$$\forall j, \quad \frac{1}{p_j}\int_{\mathbb{R}^2} \frac{\sigma(x) f_j(x)}{r_j(x)} \lambda(x) dx = \rho.$$

If ρ *is finite and if there is a finite number of server stations, all* $f \in \mathcal{F}^*$ *satisfy the above equation.*

$\longrightarrow$ optimal partionning of space equalizes traffic load !

OPTIMAL SPATIAL ALLOCATION

The extremal points of the convex set $\mathcal{F}$ are the functions $f_j = \mathbb{1}(V_j)$, where $\{V_j\}_j$ is a tessellation.

Definition 2. *The processing rates are said to be singular if there exist $j \neq k$, $C > 0$ and a Borel set A of positive Lebesgue measure such that :*

$$\forall x \in A, \; r_j(x) = C r_k(x).$$

Proposition 2. *If the processing rates are not singular, then there is an $f \in \mathcal{F}^*$ which is a tessellation. If ρ is finite and if there is finite number of server stations, all $f \in \mathcal{F}^*$ are tessellations.*

$\longrightarrow$ For CDMA, Macrodiversity does not increase the Stability Region !

REGULAR HEXAGONAL NETWORK

- Base Station on a regular hexagonal grid, with L km between two adjacent sites.
- Intensity of user arrival : λ users per square kilometers per time unit.
- A user requires to receive in mean σ bits of data from the network.
- The bandwidth is Δ Hz on a CDMA Channel.
- The attenuation function from x to y is : $K|x-y|^{-\alpha}, \alpha > 2$.

REGULAR HEXAGONAL NETWORK

$\lambda\sigma < \rho_c$ then the network is stable, if $\lambda\sigma > \rho_c$ the network is unstable,

Macrodiversity and the usual CDMA hexagonal network have the same ρ_c.

$$\rho_c \approx \frac{2}{\log(2)\sqrt{3}} \frac{\Delta}{L^2} (1 + \frac{0.94}{\alpha - 2})^{-1}.$$

$\lambda\sigma$ is the mean number of bits pumped per surface unit per time unit : this is the bit rate per surface unit of our network.

For $\Delta = 1$ MHz, $L = 1$ km, $\alpha = 4$ we get : $1, 13$ Mbits per square kilometer and per second.

ERGODIC SITES OF SERVER STATIONS

- Server Station position is an ergodic spatial point process.
- $r_j(x - Y_j)$ is a mark of the server station point process.
- Intensity of user arrival : λ and mean service requirement : σ bits.

The system is stable if $\lambda\sigma < \rho_c$ the maximal bit rate per surface unit of the system :

$$\rho_c^{-1} = \inf_{f \in \mathcal{F}} \sup_{j \in \mathbb{N}} \frac{1}{p_j} \int_{\mathbb{R}^2} \frac{f_j(x)}{r_j(x)} dx$$

$\longrightarrow$ The geometry of the network is contained in the processing rates. For TDMA and CDMA channels are function of $\frac{l(|x - Y_j|)}{I(x)}$.

ERGODIC SITES OF SERVER STATIONS

From ergodicity : ρ_c is almost surely constant.

Proposition 3. *(Under a technical condition on r_j easily satisfied in practice) If the server stations position is a Poisson Point Process, then almost surely $\rho_c = 0$.*

CONCLUSION

- The stability region defines the capacity of a network.
- For CDMA channel, under our hypothesis, macrodiversity does not increase the Stability Region on the Downlink. Similar negative results hold for macrodiversity in fixed rate CDMA channel (voice channel).
- On the contrary, macrodiversity has a huge impact on the Uplink.