An analysis of the MPR selection in OLSR

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**Plan**

**Goal**: Performance evaluation of an aspect of OLSR (a routing protocol for wireless ad hoc networks). OLSR uses a subset of nodes to broadcast control packet. We evaluate the number and the locations of this subset of nodes.

- Broadcast problems in wireless ad hoc networks
- Brief overview of OLSR (Optimized link-state routing protocol)
- MPR (Multi-Point Relay) selection algorithm
- Analysis
  - General results
  - Mean number of isolated points
  - Mean number of MPR (1st step)
  - MPR location
- Simulations
- Consequences
- Conclusion and future works
Broadcast in ad hoc networks: The storm problem

The cross node broadcasts a packet
All its neighbors receive the packet
Each neighbor broadcast the packet
The cross receives several redundant packet (equal to the number of neighbors)

The number of redundant packets received by a node of a given broadcast is equal to the number of neighbors of this node.

Such an approach is impossible if nodes are supposed to periodically broadcast control information.
Brief overview of OLSR

- OLSR is a link-state proactive ad-hoc routing protocol
- Control routing advertisements are disseminated over the network and allows the nodes to discover the network topology and compute routes
- Only a subset of nodes (MPR: Multipoint Relay) forward control packets in order to limit control traffic
  - Each node selects in its neighborhood a set of MPR which covers the 2-neighborhood
  - A node forwards a control packet if
    - It is the first time it receives it
    - It receives it from a node for which it is a MPR
**MPR selection algorithm**

- Each point $u$ has to select its set of MPR.
- Goal: select in the 1-neighborhood of $u$ ($N_1(u)$) a set of nodes as small as possible which covers the whole 2-neighborhood of $u$ ($N_2(u)$), in two steps:
  - Step 1: Select nodes of $N_1(u)$ which cover isolated points of $N_2(u)$. (That we call $MPR_1(u)$.)
  - Step 2: Select among the nodes of $N_1(u)$ not selected at the first step, the node which covers the highest number of points (not already covered) of $N_2(u)$ and go on till every points of $N_2(u)$ are covered.
MPR selection algorithm: example
MPR selection algorithm: example

First step: select nodes which cover isolated point
MPR selection algorithm: example

Second step: consider points which are not MPR and which are not covered by the selected MPR and

Select in the neighborhood, the node which cover the highest number of nodes in the two neighborhood.
Simulation results

There is approximately a reduction of 60% of redundant packets received by the nodes.
Analyze
General results

- Consider a Poisson point process with parameter $\lambda$ distributed in $B(0,2R)$.
- Two nodes $(x,y)$ are neighbors iff $d(x,y)<R$ (with $d(x,y)$ being the Euclidean distance between $x$ and $y$).
- We analyze the MPR selection algorithm for a node at the origin.
General results

Size of the neighborhood

\[ \mathbb{E}[|N|] = \lambda \pi R^2 \]
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For v in B(0,2R)\B(0,R), mean number of neighbors in B(0,R)
\[ \mathbb{E}\left[ d_v^- (v) \right] = \frac{2}{3R^2} \int_R^{2R} A(r)r\,dr = \lambda R^2 \frac{\sqrt{3}}{4} \]
General results

Size of the neighborhood
\[
\mathbb{E}[|N|] = \lambda \pi R^2
\]

For \( v \) in \( B(0,2R) \setminus B(0,R) \), mean number of neighbors in \( B(0,R) \)
\[
\mathbb{E}[d^-(v)] = \frac{2}{3R^2} \int_R^{2R} A(r)r \, dr = \lambda R^2 \frac{\sqrt{3}}{4}
\]

For \( u \) in \( B(0,R) \), mean number of neighbors in \( B(0,2R) \setminus B(0,R) \)
\[
\mathbb{E}[d^+(u)] = \lambda \int_0^{2\pi} \int_0^R (\pi R^2 - A(r))r \, dr \, d\theta
= \lambda R^2 \frac{3\sqrt{3}}{4}
\]
General results

Size of the neighborhood
\[ \mathbb{E}[|N|] = \lambda \pi R^2 \]

For \( v \) in \( B(0,2R) \setminus B(0,R) \), mean number of neighbors in \( B(0,R) \)
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\quad = \lambda R^2 \frac{3\sqrt{3}}{4} \]

Size of the 2-neighborhood
\[ \mathbb{E}[|N_2|] = 3\lambda \pi R^2 \mathbb{P}(d_0^-(v) > 0) \]
\[ = 3\lambda \pi R^2 \left( 1 - \frac{2}{3R^2} \int_R^{2R} \exp\{-\lambda A(r)\}rdr \right) \]
Probability to be an isolated point

V in $B(0,2R) \setminus B(0,R)$ is an isolated point if and only if $d^-(v) = 1$.

$$P\left(d_0^-(v) = 1\right) = \frac{2}{3R^2} \int_R^{2R} \lambda A(r) \exp\{-\lambda A(r)\} r dr$$

Probability to be an isolated point knowing that this point belong to $N_2$.

$$P\left(d_0^-(v) = 1 \mid v \in N_2\right) = \frac{P(d_0^-(v) = 1)}{P(d_0^-(v) > 0)}$$
Analysis of the first step of the MPR selection

- Points of $N_1$ which cover isolated points are chosen at the first step.
- Questions:
  - What is the number of isolated points?
  - How many MPR are selected at this step?
  - What is the proportion of $\text{MPR}_1$ vs. the total number of MPR?
  - Where are the $\text{MPR}_1$ located?

$$E[|D|] = 2\pi \lambda^2 \int_R^{2R} A(r)e^{-\lambda A(r)} rdr$$
Mean number of isolated points

Mean number of isolated points.

Ratio between the mean number of isolated points and the size of the 2-neighborhood.
Number of $\text{MPR}_1$

We give a lower and an upper bound of this number:
The upper bound is the number of isolated points.

$$\mathbb{E}[|\text{MPR}_1|] \leq \mathbb{E}[|D|]$$

The lower bound is obtained by considering a sufficient condition to belong to MPR1.

$$\mathbb{E}[|\text{MPR}_1|] \geq 2\lambda \pi \mathbb{P}(d_0^+(u) > 0) \int_0^R \int_{R+r}^{R+r} f(x, r, R) \times \exp \{-\lambda \left(2\pi R^2 - A_1(R, x, R)\right)\} \, rd^2xdr$$
A sufficient condition is: if amongst the neighbor of the blue point the farthest point of 0 is an isolated point then the blue node belongs to $\text{MPR}_1$. 
Location of the $\text{MPR}_1$

- Let be a Poisson point process in $B(o,2R)$
- We add two points, the point in $o$ for which we select the MPR set, and a point $u$ such that $d(u,o)=r$ ($r<R$).
- We give a lower and an upper bound on the probability that $u$ belongs to $\text{MPR}_1$. 
When the distance of a neighbor of \( o \) increases, it has more chance of belonging to MPR\(_1\).

MPR\(_1\) points are distributed close to the radio boundary.

Mean number of neighbors : \( \lambda \pi = 45 \)

\( R = 1 \)
The lower bound gives a very accurate approximation of the number of MPR₁.

Most of the MPR are selected during the first step of the algorithm (about 75%).
Blue points are the points of $N_2$ covered by the $\text{MPR}_1$.

Mean number of neighbors $\lambda \pi R^2 = 6$
Mean number of neighbors $\lambda \pi R^2 = 15$
Mean number of neighbors $\lambda \pi R^2 = 21$
Mean number of neighbors $\lambda \pi R^2 = 30$
Consequences (1)

- Improvements of the OLSR algorithm can only lead to similar results as the second step of the algorithm is the only one which can be changed and it selects only few nodes as MPR.

- If a MPR fails, there is a high probability (about 75%) that it is a MPR$_1$.
  - In this case, isolated points will not receive the message from this MPR.
  - It is possible the isolated points will receive the message from another path, which means it will not be the shortest path as claimed by OLSR.
Consequences (2)

A bad situation

In case of failure, a broadcast from the right cloud will not reach the other part of the network even if the network is still connected.
Conclusions

- We give analytical results on
  - Size of the neighborhood and 2-neighborhood
  - Number of isolated points
  - Number of MPR₁ and their locations
- Simulations
  - Allow us to evaluate the accuracy of the different bounds
  - Give us the proportion of MPR₁ vs. MPR
- Consequences
  - This very important number of isolated points may lead to lack of robustness in OLSR
- Future works
  - Confirm the quantitative results with other point processes and more realistic model of connectivity. For instance, we shall consider model which take into account wireless properties (802.11, CDMA, interferences, etc.).